ROYAL ASTRONOMICAL SOCIETY.

VOL. XI. *April* 11, 1851. No. 6.

J. C. ADAMS, Esq., President, in the chair.

Rev. S. K. Swann, Castle Donnington, Leicestershire; and James Walby, Esq., 13 Westbourne Park Road,

were balloted for and duly elected Fellows of the Society.

METIS.

LIVERPOOL. Equatoreal. (Mr. Hartnup.)

1851.	Greenwich M.T. h m s	R.A. h m s	N.P.D. ° ′ ″	Star of Comp.
Mar. 20	11 24 34·3	9 8 7·75	64 41 44·0	ε Leonis
	12 4 27·9	7·49	45·7	—
22	11 17 30·1	9 7 54·23	64 46 38·7	—
	11 57 23·9	54·28	42·2	—
April 3	7 54 58·0	9 9 12·92	65 27 40·9	—
	8 34 52·3	13·36	48·1	—
4	7 51 19·5	9 9 31·64	65 31 59·3	—
	8 31 13·7	31·86	68·4	—
7	8 30 30·9	9 10 37·52	65 45 46·8	—
	9 20 25·4	38·54	56·4	—
8	9 16 54·4	9 11 3·64	65 50 43·5	—
	9 56 48·9	4·55	53·9	—
23	11 4 52·9	9 20 24·43	67 15 10·5	—
	11 34 49·1	25·51	16·7	—
25	10 6 42·8	9 21 59·66	67 27 39·5	—
	10 36 39·2	60·81	47·5	—

The observations are corrected for refraction and parallax. The horizontal parallax was derived from Mr. Graham's ephemeris (*Ast. Nach.* No. 774).

The following is the assumed *mean place* of ε *Leonis* for 1851·0, derived from the Greenwich observations, 1848:—

R.A. h m s	N.P.D. ° ′ ″
9 37 23·02	65 32 32·68

On the Determination of the probable Stability of an Azimuthal Circle by Observations of Stars and of a permanent Collimator. By G. B. Airy, Esq., Astronomer Royal.

"Although the subject of the following investigations is entirely special to the Royal Observatory, and its details will therefore appear with propriety in the *Greenwich Observations*, yet I think it may not be unworthy of the notice of this Society; partly because practical problems of the same kind will occur in the management of many instruments, and partly because the problem of probabilities which occurs in it presents itself in a form which I have not seen before.

"The zero of azimuth of the horizontal circle of the Altazimuth Instrument at the Royal Observatory is determined by observing a star, and comparing the reading of the circle with the computed azimuth of the star, as calculated from the elements of the star's place and the sidereal time corrected from observations with the Transit. It has been found that this zero is unsteady to the amount of a few seconds of arc. This may depend, on the one hand, upon instability of the circle, or, on the other hand, upon the error of sidereal time in the observation (this latter error being compounded of errors in the position of the Transit-instrument, errors in the observation of transit, errors in the transmission of time from one instrument to the other, and errors in the actual observation with the Altazimuth: the elements of the star-places being sensibly perfect). Although this latter combination of causes of error appears formidable, still, from the great care employed to secure accuracy in every part, I thought it likely that their effect was extremely small; and, though in great uncertainty as to the real origin of the discordance, I was inclined to attribute it to unsteadiness of the azimuthal circle.

"In the month of February, 1850, I mounted a permanent Collimator for reference of the telescope of the Altazimuth. The object-glass of the collimator is the 25-feet object-glass of the old zenith tube; the mark is the image (as formed by a lens of short focus, diminishing the diameter to about one-sixth part) of a hole in a plate of metal, behind which is the flame of a gas-lamp. The diameter of the hole is $\frac{1}{50}$ inch, and the diameter of its image, or of the virtual mark, is therefore about $\frac{1}{300}$ inch. This mark is seen so well-defined, that its observation may be regarded as free from sensible error. It may perhaps be regarded as an unfavourable point in the mounting of this apparatus, that the object-glass is fixed in the wall of one building and the mark within another building; but every other circumstance is very favourable.

"As soon as the computation of the year's observations was completed, I collected all the observations of the collimator, and the results of computation of azimuth-zero, from February 11 to December 27, in a tabular form, under the heads of Circle-reading

for Observation of the Collimator, Apparent Azimuth of the Collimator from the Observation of Stars on the same day, and Zero of Azimuth of the Horizontal Circle from the observation of the same stars. (It is easily seen that the third quantity is equal to the excess of the first over the second; but the three quantities represent three different though connected elements, upon all which information is desirable.) An examination of these gives reason to suppose that a gradual change has taken place in each, at least in the first and second. There have also been some changes of adjustment in the intervals between lunations. For these reasons it appeared desirable, in an investigation of the comparative values of the irregular errors, not to use the absolute values of the three elements which I have mentioned, but to ascertain the mean values of these elements for each lunation, and to treat as the subjects of discussion the difference of every individual value of each element from the mean of that element for the month.

"The meanings of the different quantities of which I have last spoken can be explained in the following way:—First, we may determine the azimuth of an unknown body (as the moon) without any regard to the constancy of the circle (except for a few minutes), or to the constancy of the collimator, by observation of one or more stars. Let x be the error of azimuth produced by error in the observation of the stars, or in the transmission of time from the transit-instrument. Then the apparent azimuth of the collimator will be increased by x; and since the apparent azimuth of any object is increased by x without any alteration of the circle-reading, the zero of azimuth of the horizontal circle is increased by $-x$. Secondly, we may determine the azimuth of the moon, &c., by assuming the constancy of the circle. If the position of the circle has undergone such a change as to increase deduced azimuths by y, then the circle-reading for observation of the collimator will be increased by y; but when the observations of stars are referred to the circle, the instrumental azimuths of those stars are also increased by y, and therefore the zero of azimuth of the horizontal circle is increased by y. Thirdly, we may determine the azimuth of the moon by assuming the invariability of the collimator, using the circle only as an intermediate between the collimator and the moon, &c. Suppose the collimator to have so shifted that the deduced azimuths are increased by z. Since the circle-reading for the moon, &c., is supposed unaltered, the circle-reading for observation of the collimator must be supposed increased by $-z$; and since the circle-reading for stars is unaltered, the apparent azimuth of the collimator from the observation of stars will be increased by $-z$.

"If, now, we put p for the whole increase of circle-reading for observation of the collimator, q for that of the apparent azimuth of the collimator from the observation of stars, and r for that of the zero of azimuth of the horizontal circle from the observation of the same stars, we shall have, by collecting the quantities above found,

$$p = \quad y - z$$
$$q = \quad x - z$$
$$r = -x + y$$

The quantities p, q, and r are given numerically from the results of observation. Our object now is to determine from these the *probable values*, in a series of observations, of x, y, z, considered as chance errors, and thus to determine which of the three different assumptions used as bases for determination of the moon's azimuth (namely, 1st, that the transmission of time and the observations of stars are correct; 2d, that the circle does not change its position; 3d, that the collimator does not change its position) is most trustworthy. This is all that we can pretend to do; for, as the equations for p, q, r are not independent, it is evident that we cannot find the *absolute* values of x, y, z on any one day. But the following consideration will show that we have sufficient means for determining their *probable* values.

"Suppose it was found, in a long series of observations, that p is generally small: that is, that the errors (whatever they may be) on the second and third assumptions are nearly the same, or, in other words, that the results of the second and third assumptions always agree, that of the first being frequently discordant. Here we should not (in the ordinary familiar treatment of such matters) have the smallest hesitation in saying that we consider the first assumption as much less trustworthy than the others, provided always that we are certain that the errors (whatever they may be) of the second and third assumptions are absolutely independent. Reasoning of a similar character might be used in obtaining results on other suppositions on the relative values of p, q, r. Thus it appears that the data of the problem are sufficient to give us, in ordinary careful reasoning, a clear insight into the relative amounts of errors of the different assumptions; and the theory of probabilities is but an exhibition of such ordinary careful reasoning in a form which admits of numerical calculation.

"To put the investigation into a more technical shape, we must first remark that, so far as can be judged from the independence of the pier carrying the azimuthal circle, the two walls carrying the collimator, and the two pillars carrying the transit-instrument, the three chance errors x, y, z, are absolutely independent. Secondly, that if we put X, Y, Z, P, Q, R, for the probable values of x y, z, p, q, r, and a for a certain constant factor, we shall have $X^2 = a \cdot \Sigma x^2$, &c.; $P^2 = a \cdot \Sigma \cdot p^2$, &c.; and since, by a well-known theorem of probabilities, $P^2 = Y^2 + Z^2$, it follows that

$$\Sigma \cdot p^2 = \Sigma \cdot y^2 + \Sigma \cdot z^2$$

and similarly,

$$\Sigma \cdot q^2 = \Sigma \cdot x^2 + \Sigma \cdot z^2$$
$$\Sigma \cdot r^2 = \Sigma \cdot x^2 + \Sigma \cdot y^2$$

"These equations, however, may be proved without refer-

ence to the theory of probabilities in the following manner:— $\Sigma . p^2 = \Sigma . y^2 - 2\Sigma . yz + \Sigma . z^2$; but as y and z are strictly independent, in a long series of observations any given value of y may be supposed to be multiplied by equal positive and negative values of z, and therefore $\Sigma . yz = 0$. Thus there remains $\Sigma . p^2 = \Sigma . y^2 + \Sigma . z^2$; and similarly for the others. Having these three equations, we find $\Sigma . x^2$, $\Sigma . y^2$, and $\Sigma . z^2$, without difficulty; and thence X, Y, and Z.* I shall now proceed with the numerical elements:—

"*Mean Values, for each Lunation, of the Elements of Position for the Azimuth Circle and Collimator of the Altazimuth, from* 1850, *Feb.* 18, *to the end of the Year.*

Limits of Lunations. 1850.				Circle Reading for Observation of Collimator.	Apparent Azimuth of Collimator from Star Observations.	Apparent Zero of Azimuth of Horizontal Circle from Star Observations.
				° ′ ″	° ′ ″	° ′ ″
Feb. 18	to	March 6		190 22 21·50	179 37 59·29	10 44 21·74
March 15	—	April 4		23·08	63·97	19·11
April 17	—	May 4		25·63	64·67	20·96
May 13	—	June 3		26·69	68·59	18·10
June 13	—	July 5		34·72	78·27	16·45
July 14	—	Aug. 4		34·86	78·52	16·34
Aug. 12	—	Sept. 2		36·25	82·21	14·04
Sept. 10	—	Oct. 1		36·54	71·50	25·05
Oct. 8	—	Oct. 29		33·03	70·70	22·33
Nov. 6	—	Nov. 25		30·07	68·40	21·67
Dec. 7	—	Dec. 27		190 22 29·75	179 37 62·12	10 44 21·63

"It would appear that the collimator was disturbed between June 3 and June 13, but no intentional change in its position was made. Between September 2 and September 10 there is a considerable change of the second and third elements, evidently depending on star observations or their reductions; this is, undoubtedly, due to uncertainty on the adjustments of the transit instrument: the line of collimation of the transit was found, at the end of the year, to have changed considerably; and, after careful examination of the transits, it was concluded that the change took place *suddenly* between September 4 and September 6, and the azimuthal errors of the transit and the clock errors were investigated on that supposition; but if the change were really *gradual* (as now appears probable), that assumption of sudden error would produce an apparently sudden change in the elements of the altazimuth depending immediately on the observations of stars.

* "It is, perhaps, proper to mention, that in a paper communicated to the Society in the summer of 1850 (but subsequently withdrawn), I had treated this problem by a much more complicated process, of the correctness of which, however, I had great doubts. Upon further consideration I arrived at the simple process in the text, of the correctness of which I have no doubt."

"*Excess of each Individual Value above the Mean-Lunation-Value of the Elements of Position for the Azimuth Circle and Collimator of the Altazimuth, on every Day on which Stars were observed with the Altazimuth, from* 1850, *Feb.* 18, *to the end of the Year.*

1850.	Circle Reading for Collimator. p.	Azimuth of Collimator from Stars. q.	Zero of Hor. Circle from Stars. r.
	″	″	″
Feb. 18	+1·82	+5·53	−3·71
19	+2·29	+3·83	−1·54
20	+1·04	−0·90	+1·94
21	+1·20	+1·95	−0·75
22	−0·25	−5·82	+5·57
23	−1·57	−4·74	+3·17
26	−0·51	+0·33	−0·84
27	−0·97	−1·05	+0·08
March 3	−0·87	−1·43	+0·56
4	−1·92	+1·34	−3·26
5	−0·35	+0·38	−0·73
6	+0·06	+0·53	−0·47
15	−3·47	−5·12	+1·65
16	−0·13	+0·15	−0·28
17	−3·26	−6·47	+3·21
18	−0·75	−6·12	+5·37
21	−1·21	−0·03	−1·18
23	+0·55	+2·38	−1·83
25	+0·59	+3·30	−2·71
26	+1·00	+0·30	+0·70
27	+0·60	+1·32	−0·72
28	+2·03	+5·19	−3·16
29	+2·94	+3·42	−0·48
30	+1·35	+3·92	−2·57
31	+0·23	+0·24	−0·01
April 4	−0·44	−2·41	+1·97
17	+0·38	+2·20	−1·82
18	−0·06	+0·51	−0·57
20	−0·05	−1·75	+1·70
22	−0·22	+1·48	−1·70
23	−0·07	−1·00	+0·93
24	+2·37	−2·69	+5·06
25	−0·13	−2·38	+2·25
26	−0·43	−3·64	+3·21
April 27	−1·88	−1·01	−0·87
May 1	−2·22	+2·42	−4·64
2	+1·34	+2·21	−0·87
4	+1·02	+3·64	−2·62
13	+0·60	−0·54	+1·14
18	+1·16	+0·42	+0·74
20	+0·43	−3·17	+3·60
21	+1·26	−3·63	+4·89
22	+0·31	+0·12	+0·19
23	−0·21	−1·57	+1·36
24	+0·01	−1·81	+1·82
27	+0·76	−1·03	+1·79
28	+0·25	−6·15	+6·40
29	−0·24	−3·07	+2·83
30	−1·70	+0·83	−2·53
31	+0·43	+3·89	−3·46
June 1	−1·87	+3·84	−5·71
2	−0·59	+7·58	−8·17
3	−0·68	+4·26	−4·94
13	−0·96	−4·79	+3·83
15	−2·26	−0·27	−1·99
17	+1·01	+2·80	−1·79
18	+0·83	+2·77	−1·94
19	−0·43	−2·73	+2·30
20	+1·57	−5·23	+6·80
21	−1·37	−1·49	+0·12
22	−0·50	−0·58	+0·08
23	−1·65	−2·42	+0·77
24	−0·29	−1·46	+1·17
25	−1·86	−2·48	+0·62
29	−0·24	+6·43	−6·67
July 1	+2·21	+1·60	+0·61
2	+1·72	+1·32	+0·40
4	+1·56	+6·95	−5·39

1850.	Circle Reading for Collimator. p.	Azimuth of Collimator from Stars. q.	Zero of Hor. Circle from Stars. r.
	"	"	"
July 5	+0·60	−0·46	+1·06
14	−0·70	−3·47	+2·77
15	+1·24	−4·41	+5·65
16	+0·55	−1·18	+1·73
21	+0·64	−3·39	+4·03
22	+0·64	+2·01	−1·37
24	−0·39	−1·22	+0·83
25	+0·47	+3·56	−3·09
27	−0·28	−0·97	+0·69
29	+0·19	+2·27	−2·08
30	−0·45	+1·10	−1·55
Aug. 4	−1·91	+5·67	−7·58
12	+0·40	+1·42	−1·02
13	+0·39	−3·10	+3·49
14	−0·55	−2·58	+2·03
15	−0·35	−2·55	+2·20
17	−1·01	+4·37	−5·38
19	+0·91	−5·64	+6·55
21	−1·64	+6·51	−8·15
22	+1·20	+3·55	−2·35
23	+2·21	+4·70	−2·49
24	+1·21	+3·99	−2·78
26	−1·02	−1·81	+0·79
29	−0·55	−3·80	+3·25
Sept. 2	−1·17	−5·04	+3·87
10	+0·27	+0·98	−0·71
11	+1·11	+3·09	−1·98
12	+3·22	+5·73	−2·51
13	+1·16	−0·72	+1·88
14	+0·47	+0·96	−0·49
18	+0·94	−0·71	+1·65
21	−0·37	+2·37	−2·74
Sept. 25	−1·21	−1·40	+0·19
28	−2·08	−4·25	+2·17
Oct. 1	−3·52	−6·02	+2·50
8	+0·69	+2·39	−1·70
11	+0·43	−1·75	+2·18
12	+0·53	−1·29	+1·82
15	−0·25	−0·30	+0·05
16	+1·93	+1·63	+0·30
17	−0·04	+0·36	−0·40
18	−0·66	−0·69	+0·03
21	−1·39	+4·76	−6·15
26	−0·88	−7·76	+6·88
28	−0·58	−0·45	−0·13
29	+0·25	+3·09	−2·84
Nov. 6	−0·12	−3·30	+3·18
7	−0·41	−2·10	+1·69
8	−0·05	+1·07	−1·12
11	−0·20	+2·25	−2·45
12	−0·44	−2·27	+1·83
14	−0·28	+4·63	−4·91
16	+0·74	−1·43	+2·17
19	+0·75	−2·99	+3·74
23	+1·09	+0·12	+0·97
25	−1·03	+4·05	−5·08
Dec. 7	−0·83	+0·51	−1·34
12	+3·22	+4·23	−1·01
14	+1·93	+2·81	−0·88
17	−1·93	+0·70	−2·63
19	−0·87	+2·69	−3·56
20	−2·17	−6·35	+4·18
21	+0·64	−0·28	+0·92
27	−0·02	−4·35	+4·33

"If we square each of these numbers and form the sums of the squares for each column, they are found to be 211·55, 1422·07, and 1258·88. Thus we have the equations,

$$\Sigma \cdot y^2 + \Sigma \cdot z^2 = 211{\cdot}55$$
$$\Sigma \cdot x^2 + \Sigma \cdot z^2 = 1422{\cdot}07$$
$$\Sigma \cdot x^2 + \Sigma \cdot y^2 = 1258{\cdot}88$$

"By solution of these,

$$\Sigma \cdot x^2 = 1234{\cdot}70$$
$$\Sigma \cdot y^2 = 24{\cdot}18$$
$$\Sigma \cdot z^2 = 187{\cdot}37$$

and, remarking that the number of observations is 132, we find for the probable error of each,

$$X = \pm 2''{\cdot}07$$
$$Y = \pm 0{\cdot}29$$
$$Z = \pm 0{\cdot}80$$

"Thus it appears that the chance-error in the assumption of the invariability of the horizontal circle is little more than one-third of that in the assumption of the invariability of the collimator, and one-seventh of that depending upon the star-observations or on the transmission of time from the transit-instrument. And if we use, in combination, all the different assumptions for determining the azimuth of the moon or any unknown object, we shall have to give them weights nearly in the proportion of 50, 7, and 1.

"As the small quantity $\Sigma \cdot y^2$ is found by the expression,

$$\tfrac{1}{2}(\Sigma \cdot p^2 + \Sigma \cdot r^2 - \Sigma \cdot q^2),$$

it is obvious that a small error in either of the large numbers $\Sigma \cdot q^2$, and $\Sigma \cdot r^2$, will produce a disproportionately large effect in the ratio which $\Sigma \cdot y^2$ bears to $\Sigma \cdot x^2$ and $\Sigma \cdot z^2$. But it will not tend to remove the evidence of the strongly-marked superiority of the assumption of the steadiness of the circle above the other assumptions.

"It is also to be remarked, that the quantities which we have made the subject of discussion are not the discordances from certain fixed values, but the differences between the individual values and the mean of a certain number (generally about 12) of these same individual values. The probable errors X, Y, Z, must, for this reason, be slightly increased; but they will be all increased in the same proportion.

"The rule which is now adopted at the Royal Observatory for the reduction of the azimuth observations is, to assume the constancy of the circle through each lunation, determining its zero-point from the mean of all the results of star-observations during that lunation. It is considered that in this manner places of the moon, &c., perfectly correct (within very close limits), and strictly corresponding as regards azimuth and zenith-distance, are obtained. In the comparison of the observed places with tabular places, the comparison of both elements will be affected by the chance-error in the time-determination of that day. But upon proceeding from the stage of azimuths and zenith-distances to that of hour-angles and polar distances, the effect of this chance-error

in time will be confined absolutely to the hour-angles, and thus the injurious effects will be very much simplified, and may perhaps be effaced by further operations."

On the Measurements of Azimuths on a Spheroid.
By Lieut. A. R. Clarke, R.E.

The author commences his paper with the following words:—

"It is generally assumed in geodetical calculations, that the sum of the reciprocal azimuths of two stations on a spheroid is the same as if the stations were on a sphere and had the same latitudes and difference of longitude. This is based on Dalby's geometrical proof, that the difference between the two sums in question is very small if the stations be equally elevated above the surface. It is not, however (nor can be geometrically), shown that this difference is not greater than the probable error of observation, and therefore it may be useful to find an expression for this small difference in terms of the latitudes and longitudes of the stations, in order to see whether it may be in any case greater than the probable errors of observation, and large enough to be worth taking into account."

The author then investigates by accurate formulæ of analytical geometry, as applied to the co-ordinates of points which satisfy the spheroidal equation, the expressions for the tangents of the angles of reciprocal azimuths of two stations, and forms the accurate expression for the tangent of the sum of azimuths, and for the tangent of the excess of this sum above the sum of corresponding spherical azimuths. The expression is then cautiously reduced, and it is found, at length, that the value of this excess is insensibly small; amounting only to $0''{\cdot}000003 \times m^2 n$, where m is the number of degrees in the distances of the stations, and n the number of degrees in the difference of latitude. Then the influence of difference of heights is computed; and it is shown that, though (in cases which may arise in practice) it is greater than what has just been found, yet that it also will be insensible.

At the close of the meeting, Mr. De Morgan made some remarks upon the Gregorian Calendar, as an instrument for determining the moon's phases with sufficient accuracy to settle the question of *moonlight*. Having been led to examine it in this point of view, for the purposes of a collection of almanacs which he is preparing for publication (and which has since been published) he found that it may be made to give the day of new moon or of full moon right in three cases out of five, and with an error of only one day in almost all the other cases; the error of two days occurring only about once in 120 results. In order to obtain this amount of accuracy, the rule is:—Use the Gregorian epact to

determine full moons, and that epact increased by 1 to determine new moons; both with the well-known epact-table which appears in all extensive works or articles on the calendar.

The reason of this rule is as follows:—Clavius constructed the Gregorian Calendar expressly in such manner that the moon of his calendar should be always, as well as it could be managed, one day younger than the moon of the heavens; the object being, that the fourteenth day, by which Easter is determined, should follow the day on which the Jews keep the Passover. And as this was done with good success, it follows that one day added to the age of the calendar moon at the beginning of the year, (that is, to the Gregorian epact), gives the same degree of success to the calendar, as a means of determining the day of astronomical new moon.

If the chronological full moon had been correctly laid down, this same addition of 1 would have been equally successful as to the full moon. But the *chronological* full moon is on the *fifteenth* day of the moon. Now, half a lunation being, on the average, $14\frac{3}{4}$ days, it follows that, unless the mean new moon happen in the first quarter of its day, the mean full moon is on the *sixteenth* day; so that, in the long run, the sixteenth is the proper day three times out of four. Hence there is no occasion to increase the epact by 1, in order to determine the astronomical full moon; which is as correctly determined as the calendar will do it, by applying the existing epact to the existing hypothesis of the fifteenth day.

The preceding conclusions as to the probability of truth and error were obtained from the nineteen years 1828–1846; the following are the results for 1851, 1852, and 1853:—

New Moon.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1851	2	1	2+	1,30+	30	28+	28	26	25	24	23	22
1852	21	19+	21−	19	19	17	17	15	14−	13	12−	11
1853	10−	8	10−	8	8	6	6	4+	3	2	1,30	30

Full Moon.

1851	17	16	17	16−	15	14−	13	12−	10	10	8	8
1852	6+	5	6	5−	4−	3−	2−	1−30−	29−	28−	27−	26
1853	25	23	25	23	23−	21	21−	19−	18−	17	16−	15

Here are exhibited the days of new and full moon by the calendar: when + or − follows the date, the real day is the day after or the day before. And though in this period of three years the errors of the full moon much exceed in number those of the new moon, there is no such excess in the long run. The nineteen years 1828–1846 gave 140 cases of new moon true to the day, and 141 cases of full moon.

London: Printed by G. Barclay, Castle St. Leicester Sq.